An experimental investigation of car–trailer high-speed stability

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Abstract: Previous work on car–trailer stability has been largely limited to theoretical studies with some reference to practical experience or accident statistics. In this study, extensive and systematic experimental investigations were carried out on a combined car–adjustable-trailer system. The influence of different trailer parameters on the system high-speed stability was examined by changing the mass, dimensions, and inertial characteristics of a fully adjustable trailer. It was found that the dominant factors affecting stability were the trailer yaw inertia, nose mass (mass distribution), and trailer axle position. The tyre pressure also affects the stability, although this effect is less significant. It is interesting to see that the trailer mass alone does not dramatically affect the high-speed stability, as this runs contrary to current guidelines relating to limits on the relative mass of the car and trailer. Experimental tests on a friction stabilizer and on car electronic stability programs demonstrate that both of these improve the high-speed stability and help to delay the onset of 'snaking'.

Keywords: trailer, stability, experiment, sensitivity, stabilizer, electronic stability programmes

1 INTRODUCTION

It is estimated by the UK Caravan Club that there are over 500 000 caravans on UK roads out of a total of 30.7×10^6 road vehicles [1]. If it is assumed that the annual mileage of a caravan is one-tenth that of a car (1000 miles versus 10 000 miles per annum) it could be estimated that caravans should be involved in 0.16 per cent of accidents. In fact, current statistics suggest that caravans are involved in only 0.07 per cent of all road accidents although, when accidents do occur, the consequences for other road users are significant. It is believed that many of these accidents involve a trailer which begins to undergo 'snaking' about the hitch point to which the driver does not react or reacts improperly [2]. The divergent oscillation is often associated with a high speed and an initial impulse caused by a driver's steering input, wind gusts, uneven roads, or the passing of large vehicles.

A range of mathematical models have been developed in order to understand better the high-speed stability of towed vehicles. Bevan *et al.* [3] and Deng and Kang [4] used models with three degrees of freedom (DOFs) in their research and assumed a constant forward speed together with the yaw–side-slip degree of freedom, with the car and trailer joined at the hitch point. Anderson and Kurtz [5] developed both a 4-DOF model and a 6-DOF model that took into account longitudinal dynamics and, in the 6-DOF model, the roll dynamics of both the car and the trailer. A more comprehensive car–trailer model was developed by Fratila and Darling [6]. In this 24-DOF model the vehicle and the trailer's yaw, pitch, and roll motions were all included, and the unsprung mass vertical and spin motions were also calculated. More recently, commercial multi-body dynamics simulation software has been used in car–trailer system investigations, and Sharp and Fernandez [7] developed a highly sophisticated 32-DOF car–caravan model using AutoSim. However, despite

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the wide range of models developed, all research in this area has identified the primary factors that effect high-speed stability to be the trailer nose mass and yaw inertia.

Compared with the numerous simulation-based theoretical studies, publications relating to the experimental analysis of car–trailer stability are very limited. Although a small number of publications presented limited measurement results to validate their simulation work [5, 6], none has presented rigorous experimental results to demonstrate the effect of different trailer parameters on the system stability. A few car original equipment manufacturers and caravan organizations have performed experimental measurements; however, much of this work remains unpublished. In reality, most of the practical knowledge in this field is still limited to experience or accident statistics.

In this paper, extensive experimental investigations on the high-speed stability of car–trailer systems are described. A fully adjustable trailer in which the mass distribution and trailer dimensions could be altered was used in order to avoid the difficulty of parameter interactions associated with fixed caravan structures. By changing only one parameter at a time, it was possible to examine the effect of individual factors affecting stability. In addition, devices intended to enhance the system high-speed stability, e.g. trailer stabilizers and vehicle electronic stability programs (ESPs), were also investigated.

2 TEST EQUIPMENT AND PROCEDURE

2.1 Towing vehicle

As estate cars are among the most common vehicles towing caravans or trailers, a Ford Mondeo Estate 1.8T was used in this study. Table 1 lists the specifications of the vehicle.
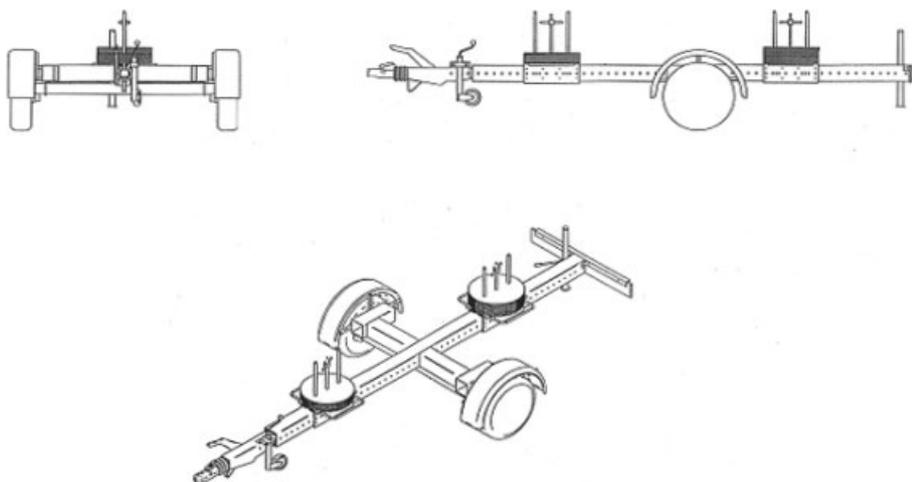
2.2 Adjustable trailer

To emulate a wide range of caravan settings, an adjustable trailer was designed as shown in Fig. 1. It consisted of a central steel column mounted on a standard torsional rubber caravan suspension. Two mass cradles sat on either side of the axle: one in front and one behind. These cradles were moved along the trailer and secured in place with a vernier set of holes and location pins. Each cradle was loaded with masses of up to 500 kg in increments of 10 kg. The nose of the trailer could be extended to increase the distance between the tow hitch and the axle. Again pins were used to fix this extendable column in place.

This adjustable trailer provided great flexibility in configuring different arrangements including wide ranges of masses, yaw inertias, nose masses, and lengths. Another advantage of this adjustable trailer was that a single parameter such as the yaw inertia could be changed while maintaining constant values for the other parameters (mass, nose mass, and tow bar length). This overcame the main weakness of using production caravans or trailers in that the

Table 1 Ford Mondeo Estate 1.8T specifications

Engine	1.8-l turbo diesel
Transmission	Five-speed manual
Kerb mass (kg)	1473
Permissible gross mass (kg)	2005
Maximum tow ball mass (kg)	75
Wheelbase (m)	2.7
Wheel track (m)	1.53
Tyres	195/60R15

Fig. 1 Adjustable trailer

parameters are not independently variable. It should be noted that the wheel track and the height of the trailer could not be changed.

The geometry and mass distribution of a trailer can be easily measured. However, the yaw inertia, one of the most important factors affecting towing stability, had to be measured on a turntable device developed for this purpose. This consisted of a steel frame supported on four air bearings that rotated about the central axis. Six linear coil springs were used to provide a known torsional stiffness about the central axis of rotation. This design ensured low values of rotational friction and, by measuring the period of oscillation, the yaw inertia of the trailer was estimated. The principle of trailer inertia measurement is presented in Appendix 1.

A Bailey Discovery 2000 was chosen as the baseline caravan. It had the same wheel track as the adjustable trailer and the trailer could be configured to the same length, mass distribution, and yaw inertia as the unladen baseline vehicle. There is no unique trailer layout that will satisfy the dimensional mass and inertia parameters and so a method was devised to obtain a possible solution (Appendix 2). The final settings of the adjustable trailer are listed in Table 2 and there is good agreement between the baseline caravan and the trailer. The complete car–trailer combination is shown in Fig. 2.

2.3 Instrumentation

A number of signals were measured on the test vehicle and trailer. The most important signals were the car–trailer articulation angle, the vehicle speed, and the driver steering input. In addition, the vehicle and trailer yaw rate, the lateral acceleration, and the longitudinal acceleration were also recorded in each test. All these signals were logged using a mobile data acquisition system from Race Technology. The sampling frequency was set to 100 Hz in the test. Figure 3 shows the overall instrumentation scheme.

3 EXPERIMENTAL DESIGN AND ANALYSIS

3.1 Test procedure

After setting up the trailer with the principal parameters as outlined in Table 2, extensive road tests were carried out to study the various factors that may affect the stability of the car–trailer system. The vehicle was driven in a straight line at various speeds from 30 mile/h to 60 mile/h. The snaking of the trailer was initiated by an impulse steering input from the driver. It would have been preferable to use a steering robot for these tests but this was not available. Therefore, in order to minimize driver error, each test was repeated at least three times.

Table 2 Comparisons of the caravan and adjustable trailer settings

	Value for the following	
	Caravan	Adjustable trailer
Total mass (kg)	879.6	877.2
Nose mass (kg)	51.3	52.4
Tow-hitch-to-axle distance (m)	3.77	3.77
Centre-of-gravity-to-axle distance (m)	0.22	0.23
Yaw inertia about its centre of gravity (kg m^2)	2601	2612
Wheel track (m)	1.9	1.9
Tyre	195/70R14	195/70R14

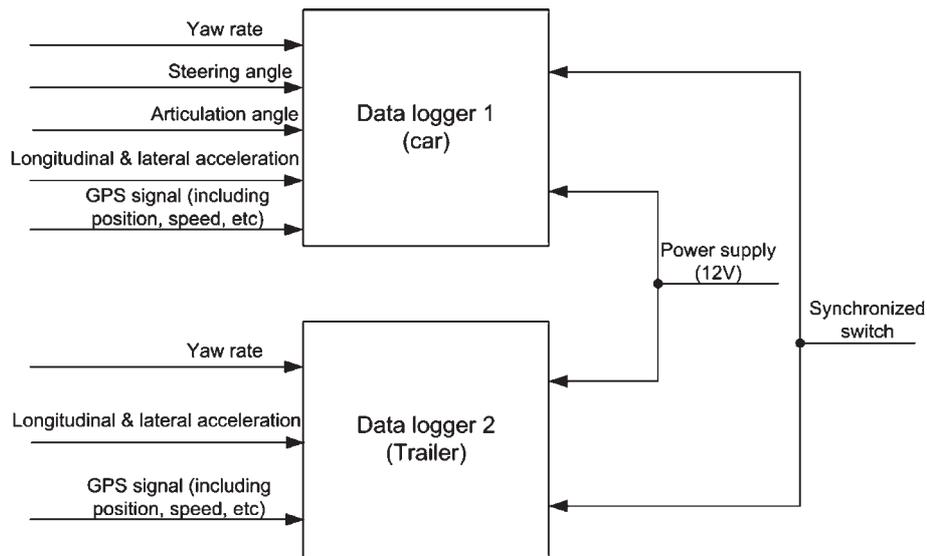**Fig. 2** Car–trailer towing system

Fig. 3 Instrumentation scheme (GPS, global positioning system)

Figures 4 and 5 show typical results at 30 mile/h and 55 mile/h respectively. It can be seen that, when the vehicle speed increases, the system becomes less stable and the trailer oscillation takes a longer time to settle down.

3.2 Data analysis

In order to quantify the car–trailer behaviour it would be possible to present data relating to the car–

trailer relative angle and phase delay and the car and trailer absolute position and angle. However, British Standard BS AU 247:1993 (ISO 9815:1992) [**8**] has been developed specifically for the analysis of towed vehicle stability and was therefore felt to be most appropriate for the 600 tests undertaken in this study. This standard uses the damping ratio of the car–trailer oscillation to evaluate the lateral stability of the system. A damping ratio of unity indicates no oscillation, while a damping ratio of zero indicates no decay and constant amplitude. If the damping ratio
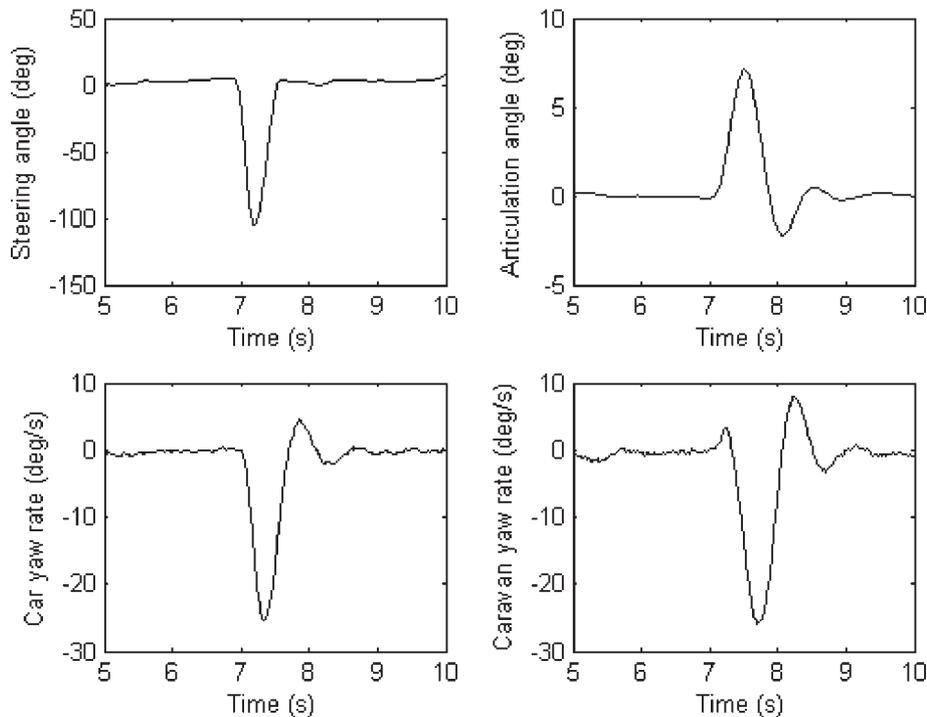

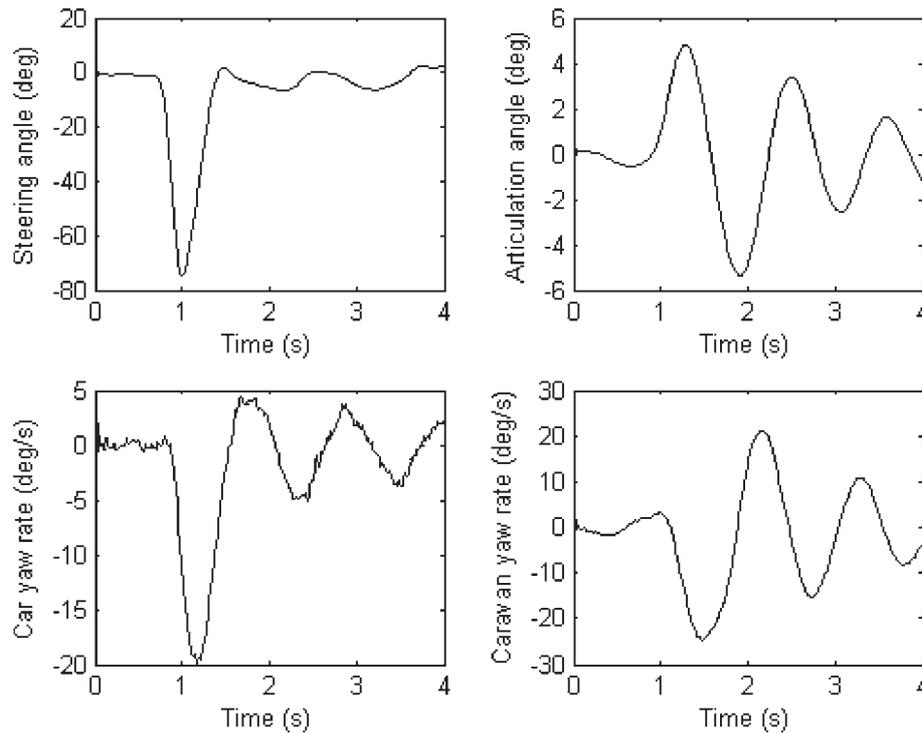**Fig. 4** Baseline trailer testing results at 30 mile/h (steering input at 7 s)

Fig. 5 Baseline trailer testing results at 55 mile/h (steering input at 0.7 s)

is negative, this indicates an increasing amplitude of oscillation and instability. In the following analysis, the damping ratio is used to investigate the effect of various parameters on the system stability.

The damping ratio ζ is calculated as

$$\zeta = \frac{\ln \chi}{\sqrt{\pi^2 + (\ln \chi)^2}} \quad (1)$$

where χ is the mean value of the amplitude ratio and is calculated using

$$\chi = \frac{1}{n-2}\left(\frac{A_1 + A_2}{A_2 + A_3} + \frac{A_2 + A_3}{A_3 + A_4} + \cdots + \frac{A_{n-2} + A_{n-1}}{A_{n-1} + A_n}\right) \quad (2)$$

where A_i is the oscillation amplitude of the articulation angle and n is the number of oscillations considered.

Each high-speed manoeuvre was repeated at least three times in order to reduce the effect of external disturbances such as road and wind variations and to average the effect of variable steer inputs and variable vehicle speeds. As an indication of the degree of variability in the test results, Fig. 6 is presented. The three data points associated with each speed measurement are presented, together with the average value. Erroneous data points, such

as that at 28 mile/h with $\zeta = 0.495$, were omitted from the calculation of the average damping value. This was considered appropriate given that the variations could be due to a combination of steer, speed, road, and wind effects. It is apparent that experimental scatter is more evident at a low speed where the damping ratio is higher, the relative angle peaks are less distinct, and the signal-to-noise ratio is less good. It could be argued that experimental error during periods of relative stability is less of a concern than error during periods of instability. If these tests were to be repeated, experimental error could be reduced by undertaking measurements on a smooth road surface, with little atmospheric wind, using a steering robot and precise vehicle speed control.

3.3 Comparison of baseline caravan and trailer

A back-to-back test was carried out on the baseline caravan and the equivalent trailer (Table 2). Figure 7 shows the damping ratios of the two systems at different speeds. In general, the damping of the trailer was similar to that of the caravan, although at high speeds (over 50 mile/h) the agreement was less good. One factor that could contribute to this is the different aerodynamics of the two systems. The caravan has a larger frontal area and, as a result, more significant aerodynamic drag forces. This
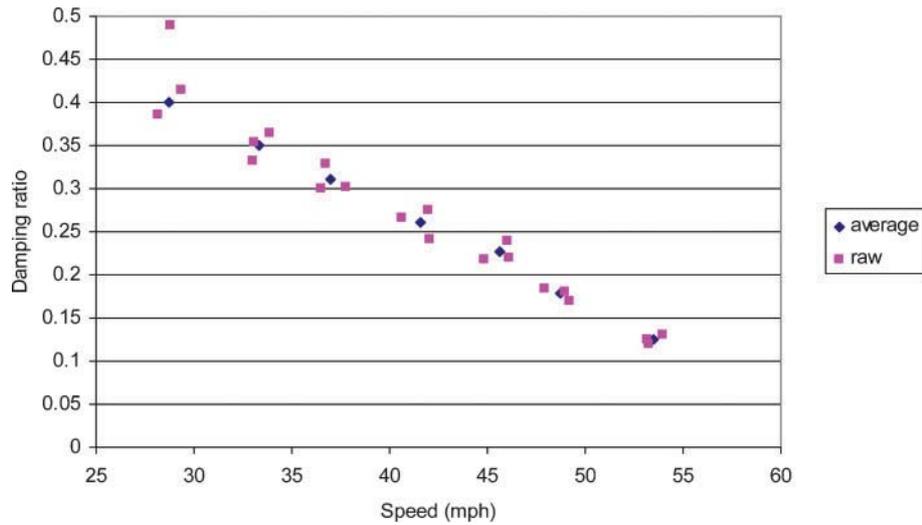
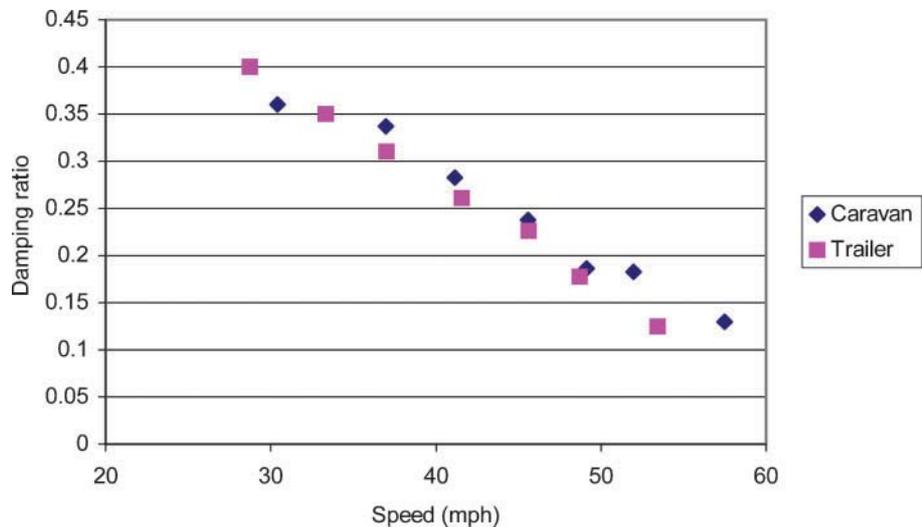**Fig. 6** Typical experimental data

Fig. 7 Damping versus speed for the baseline caravan and the standard trailer

increases the tension force in the caravan tow bar and results in a more stable system. A detailed study on caravan aerodynamics performed by Darling and Standen [**9**] showed that the aerodynamics of car–caravan systems are affected by both the car and the caravan. While recognizing that aerodynamics will be important at high speeds, it is argued that the adjustable trailer demonstrates important trends that nonetheless help to explain the high-speed behaviour. It was concluded that, despite the small differences between the trailer and baseline caravan, the trailer was a good experimental tool for research into caravan high-speed stability.

3.4 Trailer parameter sensitivity study

After establishing the baseline trailer characteristics (Table 2), a sensitivity study on various trailer parameters was carried out. The aim of this experimental study using the adjustable trailer was to alter one parameter at a time in order to assess the sensitivity of this complex dynamic system to single variables. In practical applications, in a real caravan, it is likely that many important parameters are interrelated. However, the complexity of the combined car–caravan system makes it difficult to establish the significance of individual factors if several are changed at once.

3.4.1 Nose mass

The nose mass of a trailer is very sensitive to loading, and it is well known that high-speed stability is highly influenced by this parameter. In the first study the nose mass of the trailer was varied while other settings, e.g. total mass, yaw inertia, and

wheel-to-tow hitch distance, remained unchanged. Table 3 lists the different nose mass settings and the ratios of each nose mass to the overall trailer mass. Figure 8 shows the corresponding damping versus speed plots and Fig. 9 shows the estimated speed when the damping ratio becomes zero. It was found that an increased nose mass improved the system

stability, although the improvement becomes less significant when the nose mass rises above 6–7 per cent of the total weight. This effect is thought to be caused by the increase in the car rear axle load associated with nose mass that increases the ability of the tyres to generate side forces that damp out the oscillation. In addition, the increased nose mass raises the tow ball friction, which in turn helps to damp out the oscillation. However, it should be noted that there are limits on the maximum nose mass, partly as a result of structural strength issues and partly because too much nose mass will reduce the front axle load and worsen the handling performance of the vehicle.
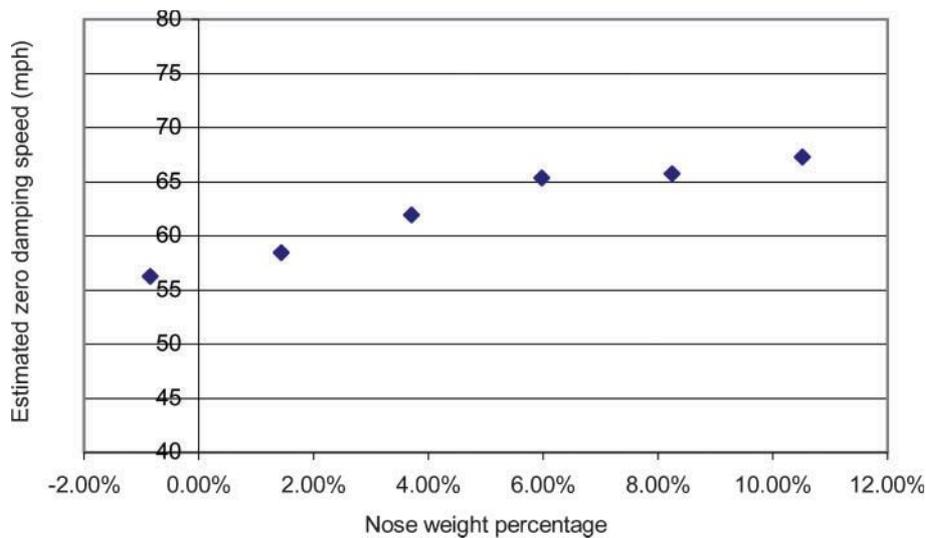
Caravan users often prefer a light nose mass as it is easier to manoeuvre the caravan and to couple it to

Table 3 Trailer nose mass settings

Test	Nose mass (kg)	Nose-mass-to-trailer-mass ratio (%)
1	−7.4	−0.84
2	12.5	1.43
3	32.5	3.70
4 (standard)	52.4	5.97
5	72.3	8.24
6	92.2	10.51
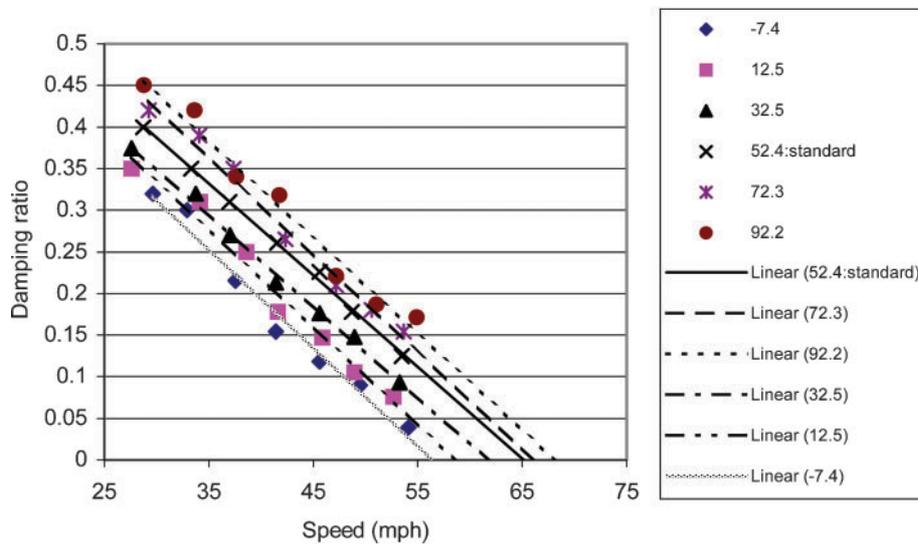
Fig. 8 Damping versus speed for the different nose masses (kg) given in the key

Fig. 9 Zero damping speed versus percentage nose weight

the car. However, this could be detrimental to the high-speed stability as demonstrated by the experimental results. For instance, with the car–caravan combination investigated here, it is shown in Fig. 9 that a nose mass of less than 2 per cent of the overall trailer mass has a zero damping speed below 60 mile/h. This could be very dangerous if the caravan were towed on motorways. In order to have a safe speed margin, it is recommended that the caravan should have a nose mass of around 6–8 per cent of the overall caravan mass.

Commercially available nose mass gauges (weighing scales) are available and these are widely used. In order to adjust the nose mass it is recommended that heavy items are positioned on the floor above the caravan axle and moved small distances in order to shift the vehicle's centre of gravity. In this way the nose mass can be changed without significantly increasing the yaw inertia.

3.4.2 Trailer yaw inertia

Following the nose mass testing, the trailer yaw inertia was varied while maintaining the other parameters in the same configuration as the initial setting. Tests were repeated and the corresponding system responses are shown in Fig. 10. It can be seen that, when the trailer inertia increases, the damping of the combined car–trailer system decreases dramatically. The inertia effect suggests that, when a driver is loading a caravan, the mass should be placed as close to the centre of gravity as possible in order to minimize the resulting increase in inertia.
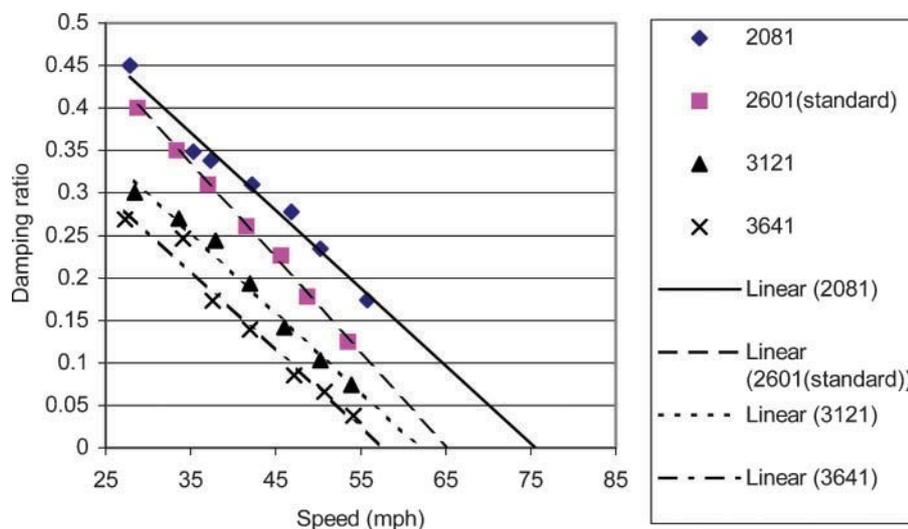
3.4.3 Trailer mass

To study the effect of trailer loading on the system stability, the trailer was set to different masses while maintaining other settings at the baseline level (Table 2). For example, an increase in trailer mass was accompanied by a reduced separation of the individual trailer mass elements so that the yaw inertia and nose weight remained unchanged.

Figure 11 shows the damping ratio versus speed plots for various trailer masses. The trailer mass in isolation, given a constant yaw inertia, did not have a significant effect on the high-speed stability. It can be concluded that the reduced stability of heavy trailers is primarily associated with an increase in yaw inertia rather than the mass in isolation. Only by varying one parameter at a time was it possible to isolate this effect.

3.4.4 Trailer axle position

In addition to the load, inertia, and mass distribution, the distance from the tow hitch to the trailer axle was also investigated. Tests were performed with various length settings while other parameters remained the same as the baseline configuration. It was found that, the longer the distance from the tow hitch to the tyre contact patch, the more stable is the system, as shown in Fig. 12. This can be attributed to the fact that, when the towing length increases, the trailer lateral tyre forces act on a larger lever arm with respect to the tow hitch and this helps to stabilize the trailer oscillation.
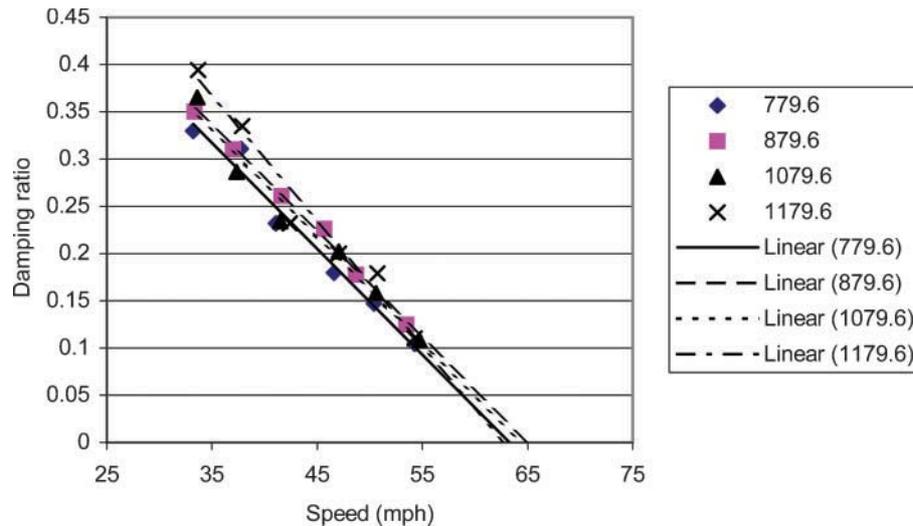

Fig. 10 Damping versus speed for the different trailer inertias (kg m^2) given in the key

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Fig. 11   Damping versus speed for the different trailer masses (kg) given in the key
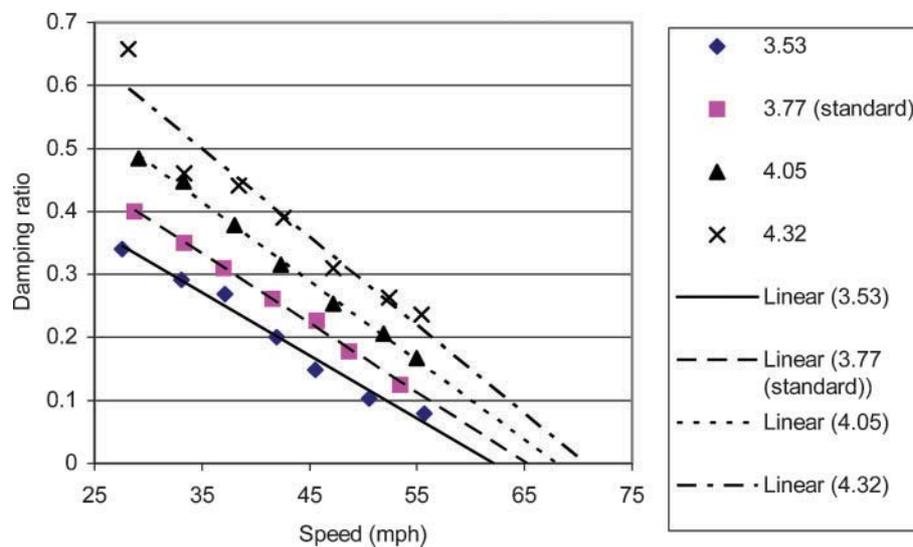

Fig. 12   Damping versus speed for the different trailer towing lengths (m) given in the key

3.4.5   Tyre pressure

Since the tyre pressure will affect the vehicle dynamic response, tests were repeated at two different tyre pressure settings for both the car and the trailer. Figure 13 shows the corresponding damping ratio versus speed plots. In general, the higher the tyre pressure, the larger is the damping, although the change is not very significant.

### 3.4.6   Observations from the parameter study

The car–caravan relative angle damping ratio is a useful tool that identifies the parameters that influence high-speed stability. The experimental results presented here were used to validate a simulation study undertaken by the present authors. There was good agreement obtained for each of the parameter variations and the findings are in line with those previously documented in simulation work conducted by others [7].

3.5   Devices to enhance trailer stability

3.5.1   Trailer stabilizer

To improve the trailer stability, various stabilizer devices are commercially available. Among them, friction stabilizers are the most common. Figure 14 shows the effect of a stabilizer for a less stable trailer setting. It was found that, although the stabilizer is beneficial and increases the damping ratio,
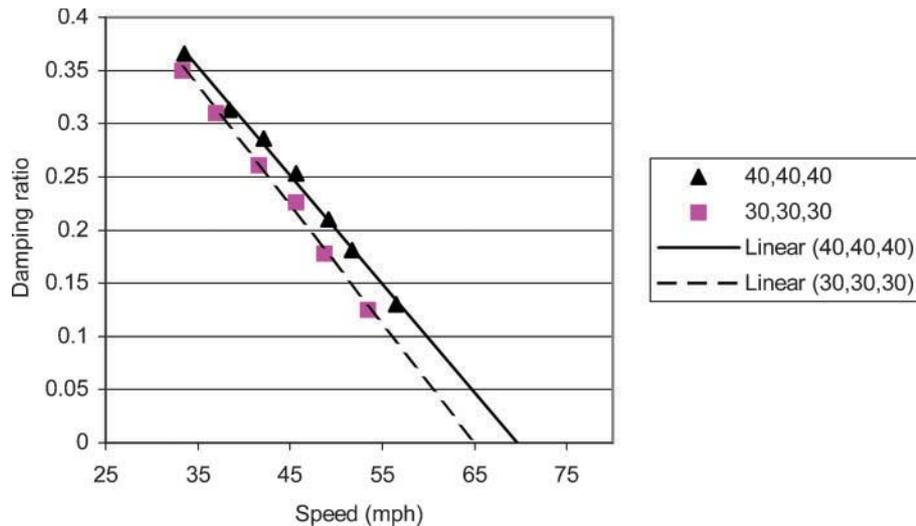
Fig. 13 Damping versus speed for the different tyre pressures (lbf/m^2) given in the key
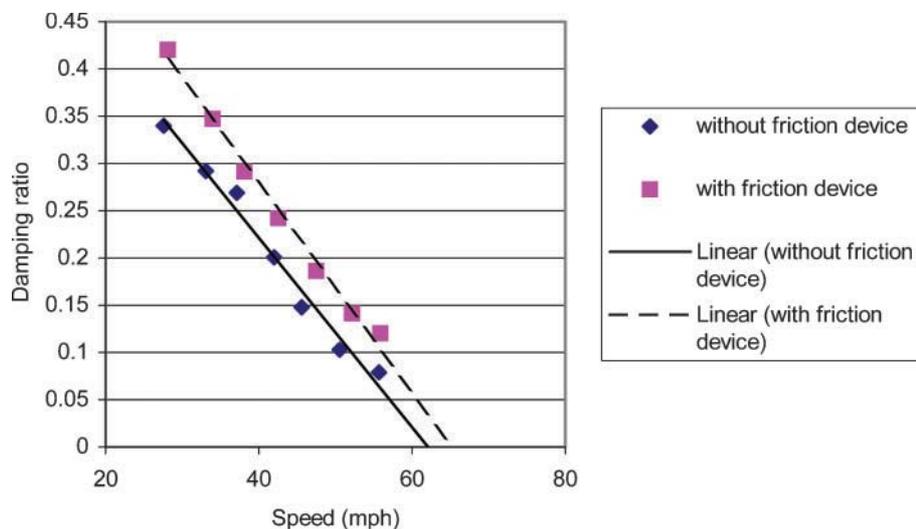
Fig. 14 Damping versus speed with and without the friction stabilizer

the improvement is limited. In this instance, the zero damping speed increased from 61 mile/h to 66 mile/h. This suggests that a driver should still drive cautiously at a sensible speed even if a stabilizer is fitted. The findings here are supported by the simulation study conducted by Sharp and Fernandez [7], which concluded that a friction stabilizer can only provide limited benefits to trailer stability.

3.5.2 Electronic stability programs

First introduced by Bosch in 1995, ESPs are widely offered on European cars and are gaining a share of the market in the USA. In 2003, nearly 20 per cent of newly produced cars in the UK were fitted with ESPs and in Germany this figure reached 55 per cent. ESPs use the components of the anti-lock brake system

(ABS) and the traction control system (TCS) to influence the handling dynamics during high-speed and high-acceleration conditions. Apart from the sensors already employed in ABS and TCS systems, they also employ a steering-wheel angle sensor, a yaw rate sensor, a lateral acceleration sensor, a pressure sensor, and a control algorithm to identify the vehicle-handling state. If it is established that the vehicle dynamic response differs from that expected, beyond a threshold limit, the ESP system has the ability to brake individual wheels without driver intervention and/or to vary the brake force when the driver is braking. It assists the driver in situations approaching the limit-handling condition of the vehicle and can greatly improve the vehicle stability.

Despite the performance improvements provided by ESPs, nearly all ESP systems are designed for

vehicles alone and take no account of external loads such as towed vehicles. Although some major car companies have conducted experiments in this area, the results have not been published and it is not clear to what extent ESP systems aid stability when towing.

In this study the present authors tested a number of new cars, concentrating especially on those where the ESP could be switched on and off. As the ESP is a safety critical device, it will not operate in normal driving conditions. Thus, to maximize the intervention of an ESP on the trailer snaking, the adjustable trailer was set to a very unstable configuration with a large yaw inertia and zero nose mass. Tests were performed on several vehicle models and the results

were positive, although, depending on the ESP control algorithm and threshold level for intervention, they provided different levels of improvement. Figures 15 and 16 demonstrate the effect on the trailer oscillation at a speed of 47 mile/h for one of the test vehicles with ESP. The intervention of the ESP can be clearly seen from the car longitudinal acceleration in Fig. 16. By comparing the car and trailer yaw rates with and without the ESP system in operation, it is clear that the oscillation decay is more rapid with the ESP system working, and this should help to improve the system stability and safety. Figure 17 shows the damping ratio with and without the ESP at various vehicle speeds. At low vehicle speeds the ESP does not intervene and the
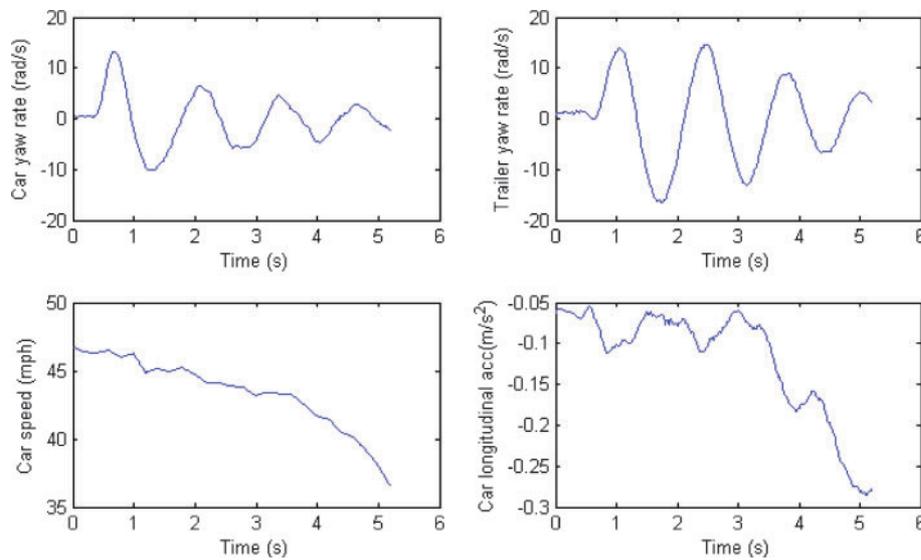

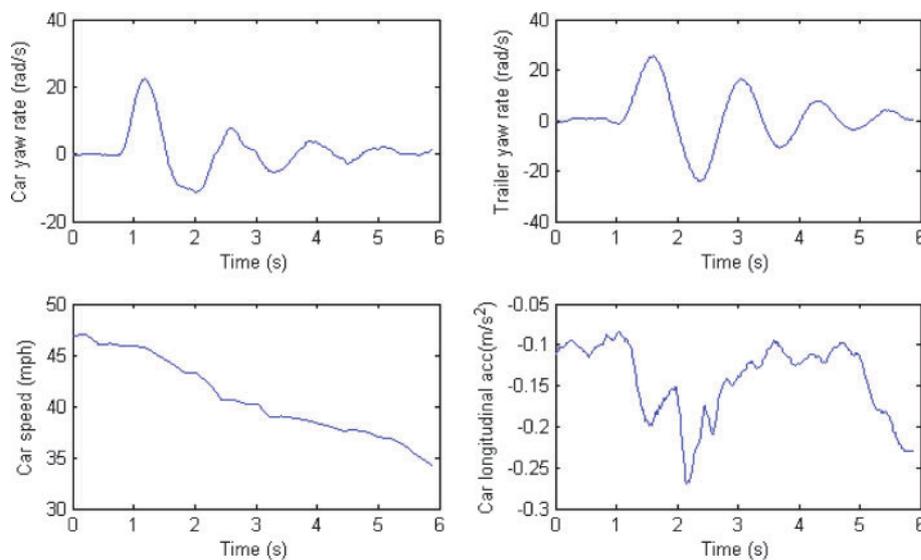Fig. 15   Experimental results at 47 mile/h with the ESP off

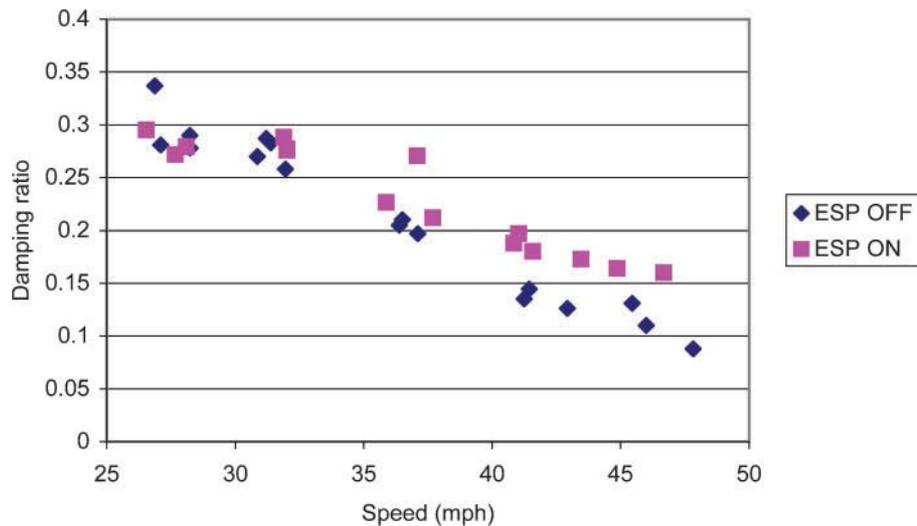Fig. 16   Experimental results at 47 mile/h with the ESP on

Fig. 17   Damping versus speed, showing the influence of the ESP on stability

two systems show no clear difference. However, as the vehicle speed increases, the snaking of the trailer becomes more violent and forces the towing vehicle off its intended path. The ESP system is activated and tries to stabilize the vehicle, thereby stabilizing the trailer. Owing to the intervention of ESP the damping ratio curve levels off at high vehicle speeds, and very low or zero damping is avoided.

4   CONCLUSIONS

Very little work has been published on the experimental measurement of high-speed car–trailer stability. In this study, extensive experimental measurements were carried out on a combined car–adjustable-trailer system. By adjusting the trailer settings, the effect of different trailer parameters on the system stability was examined. It was found that the dominant factors affecting stability were the trailer yaw inertia, nose mass (load distribution), and trailer axle position. The tyre pressure also affects the stability, although the effect is less significant. It is interesting to see that the trailer mass alone does not dramatically affect the stability; however, as a heavier trailer normally has a larger yaw inertia, a limit should be placed on the relative car–trailer masses.

A friction stabilizer is shown to be helpful in improving the system stability, although in these tests the stability was not increased hugely. In addition, high-speed towing tests were carried out on cars fitted with an ESP which automatically brakes individual wheels and controls the engine throttle position should the vehicle dynamic response differ from that expected. These tests demonstrated that,

if the dynamic response 'error' exceeded a preset threshold level, the ESP operated and the high-speed stability was improved by controlling the car yaw oscillation associated with trailer instability.

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APPENDIX 1

Trailer inertia measurement

By measuring the period of oscillation of a torsionally sprung rotating turntable, the yaw inertia of a trailer can be estimated. As the first step of laboratory testing, the yaw inertia of the turntable was measured. The table was turned to a predefined angle and then released. The period T_{table} of oscillation was measured. The natural frequency ω_{table} of the rotation is

$$\omega_{table} = \frac{2\pi}{T_{table}} \tag{3}$$

The yaw inertia of the turntable is

$$I_{table} = \frac{k_{torsion}}{\omega_{table}^2} \tag{4}$$

where $k_{torsion}$ is the torsional stiffness of the turntable, determined by experimental measurement.

The trailer was then placed on the turntable with the wheel centre above the central pivot axis. The same procedure was applied to measure the period of the combined system response. Then the yaw

inertia I_{total} of the combined system was determined. Therefore the yaw inertia of the trailer about its axle is

$$I_{trailer,axle} = I_{total} - I_{table} \tag{5}$$

Using the parallel axis theorem, the yaw inertia of the trailer about its own centre of gravity is

$$I_{trailer,CoG} = I_{trailer,axle} - M_{trailer}X_{CoG}^2 \tag{6}$$

where $M_{trailer}$ is the overall trailer mass and X_{CoG} is the distance of the trailer centre of gravity to the wheel centre, which can be calculated from

$$X_{CoG} = \frac{M_{nose}Y}{M_{trailer}} \tag{7}$$

where M_{nose} is the trailer nose mass, which can be easily measured, and Y is the distance from the tow hitch to the trailer axle.

APPENDIX 2

Trailer adjustment

The critical dimensions for the adjustable trailer are presented in Fig. 18.

The procedures to adjust the trailer setting were as follows.

1. Adjust the hitch length Y of the trailer.
2. Measure the unladen trailer overall mass $M_{unladen}$, centre-of-gravity position $X_{unladen}$, and yaw inertia $I_{unladen}$ using the method described in Appendix 1.
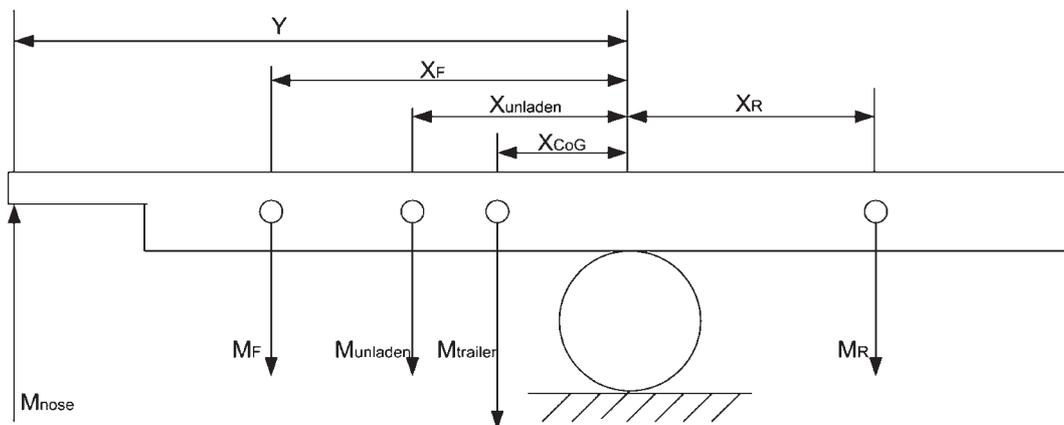
Fig. 18 Trailer geometry

Table 4 Trailer parameters (without cradle mass)

Total mass (kg)	496.4
Nose mass (kg)	55
Tow-hitch-to-axle distance (m)	3.77
Centre-of-gravity-to-axle distance (m)	0.42
Yaw inertia about its centre of gravity (kg m^2)	1281

3. Place the necessary masses at the front and rear of the trailer and adjust the mass to the proper position. The calculations of the masses required and its position are as follows.

The unladen trailer parameters are listed in Table 4.

The overall trailer mass is

$$M_{\text{trailer}} = M_{\text{unladen}} + M_F + M_R \tag{8}$$

where M_{unladen} is the unladen trailer mass as listed in Table 4, and M_F and M_R are the front and rear laden masses.

The moment equation about the trailer axle is

$$M_{\text{trailer}} X_{\text{CoG}} = M_F X_F - M_R X_R + M_{\text{unladen}} X_{\text{unladen}} \tag{9}$$

where X_F and X_R are the distances from the trailer axle to the front and rear cradles respectively, and X_{unladen} is the unladen trailer centre-of-gravity position relative to the axle.

The yaw inertia of the trailer about its axle is

$$I_{\text{trailer,axle}} = I_F + I_R + I_{\text{unladen}} + M_F X_F^2 + M_R X_R^2$$
$$+ M_{\text{unladen}} X_{\text{unladen}}^2 \tag{10}$$

where I_{unladen} is the unladen trailer yaw inertia about its own centre of gravity obtained from measurement; I_F and I_R are the inertia of the front and rear mass about their own centre of gravity respectively and can be calculated as

$$I_F = 0.5 M_F r^2 \tag{11}$$

$$I_R = 0.5 M_R r^2 \tag{12}$$

where r is the radius of the plates.

Combining equations (4), (5), and (3) gives

$$I_{\text{trailer,axle}} = 0.5 M_F r^2 + 0.5 M_R r^2 + I_{\text{unladen}} + M_F X_F^2$$
$$+ M_R X_R^2 + M_{\text{unladen}} X_{\text{unladen}}^2 \tag{13}$$

Combine equations (1), (2), and (6) together. There are four unknowns M_F, M_R, X_F, and X_R. If any one of these is predefined, the rest can be solved. For a given setting, there is no unique solution and therefore it is reasonable to set one of the cradle masses and to solve for the associated positions.